Dynamic Responses of Solid Rockets during Rapid Pressure Change

S. L. TURK,* R. A. BATTISTA,† K. K. KUO,‡ L. H. CAVENY,§ AND M. SUMMERFIELD,¶
Princeton University, Princeton, N. J.

Solid rocket performance during rapid pressure excursions differs greatly from predictions based on steady-state burning rate data. Rapid pressurization (150–250 kpsi/sec) following a sudden throat area decrease in a low L^* combustor produces pressure overshoots of 10% and indicated burning rate overshoots in excess of 50%. A transient internal ballistics model was developed incorporating nonsteady continuity and energy equations for the chamber, nonsteady energy equation for the propellant condensed phase, and a modified Zeldovich heat feedback function for the propellant (which for the conditions considered is known to burn with a thin quasi-steady reaction zone). Sensitivity analyses using the model indicate that accurate surface temperature and temperature sensitivity data are needed. With reasonable estimates of surface reaction zone temperature and measurements of temperature sensitivity of burning rate, good agreement between the measured and the calculated p vs t was found for a nonmetallized composite propellant in a low L^* combustor. High pressure exponent, high temperature sensitivity of burning rate, high dA_t/dt, low burning rate, and low L^* prominently increase the dynamic effects.

Nomenclature

A_b = propellant burning surface, cm^2
A_t = throat area, cm^2
c = specific heat, cal/g°K
c_p = specific heat of combustion gases at constant pressure, cal/g°K
E_s = activation energy in pyrolysis law, cal/g-mole
L^* = ratio of chamber volume to throat area, V_{ch}/A_t, cm
m = mass flow rate, g/sec
M_w = average molecular weight, g/g-mole
n = exponent of ap^n burning rate law
p = pressure, atm
r = burning rate, cm/sec
R = universal gas constant, 1.98 cal/g-mole °K
t = time, msec, sec
T = temperature, °K
V = chamber volume, cm^3
x = distance from burning surface, cm
α = thermal diffusivity, cm^2/sec
γ = ratio of specific heats
λ = thermal conductivity, cal/cm °K sec
ϕ = temperature gradient at interface between surface reaction zone and nonreacting condensed phase, °K/cm
ρ = density, g/cm^3
σ_p = temperature sensitivity of burning rate at constant pressure $(\partial \ln r/\partial T_0)_p$, °K^{-1}
τ = characteristic times, sec

Subscripts

0 = initial propellant conditions
1 = equilibrium condition at onset of throat area change
2 = equilibrium condition of new throat area
app = apparent
b = burning propellant
c = condensed phase
ch = chamber
f = flame zone
eq = steady-state condition which corresponds to instantaneous pressure
if = interface between the very thin surface reaction zone and the nonreacting condensed phase
max = conditions at maximum overshoot
n = nozzle
ref = reference condition
s = surface reaction zone; gas/burning-surface interface

Introduction

THE measured performance of solid rocket motors during rapid pressure excursions (such as occur during rapid ignition, variable throat area operation, and combustion instability) differs greatly from predictions made using steady-state burning rate data and simple transient mass balance solutions. Studies of these dynamic effects are frequently limited by incomplete gas dynamics and combustion models and by difficult-to-interpret data. In the continuing effort to reduce these limitations, this study deals with theoretical and experimental aspects of the dynamic response of burning rate, chamber pressure, flame temperature, and chamber temperature during rapid pressure changes (as large as 250 kpsi/sec). The importance of these dynamic effects is dependent on the type of rocket motor (or solid propellant application) being considered. For example, the rapid pressure rise following either a hard ignition or a rapid decrease in the discharge area of a variable throat area nozzle can produce burning rate overshoots in excess of 50% and pressure overshoots 20% above equilibrium conditions. Also, there are situations where the dynamic burning responses are reinforced by the dynamic chamber responses to produce nonacoustic instabilities (e.g., L^* instability).

Two opposing combustion effects cause the instantaneous burning rate to differ from the steady-state burning rate at the corresponding instantaneous pressure. These are 1) the nonsteady thermal profile in the condensed phase, and 2) the out-of-phase blowing effect of the reactive gases leaving the burning surface.[1] As an example, we will consider the situation of decreasing the throat area to produce an increase in pressure under controlled conditions. At the lower burning rate, corresponding to the lower pressure prior to pressurization, the thermal wave penetrates further into the propellant than at the higher pressure. During pressurization, the burning rate increases, and the transition to a thinner thermal wave occurs. Since the propellant is in effect preheated, the burning rate is enhanced while the pressure is increasing and the overheated

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* Undergraduate Student; presently Graduate Student, Massachusetts Institute of Technology. Student Member AIAA.
† Graduate Student. Student Member AIAA.
‡ Research Associate. Member AIAA.
§ Senior Member of Professional Staff. Associate Fellow AIAA.
¶ Professor of Aerospace Propulsion. Fellow AIAA.

surface layer is being burned out. The preheating effect is partially countered by the increased blowing in the flame zone, which tends to decrease the heat feedback from the flame when the burning rate exceeds the equilibrium burning rate. The interactions resulting from the rapid changes in pressure also will affect the flame temperature and chamber gas temperature, which will affect, in turn, the mass discharge rate through the nozzle.

In previously published studies, either the interactions between the propellant combustion and the chamber were not considered (e.g., steady state burning rate laws were used) or the burning model required gas phase properties which have not been measured. In this study, the heat feedback from the flame zone and surface reactions is based on the Zeldovich approach which does not require detailed information on the gas phase and surface reactions. The Zeldovich approach has been extensively developed in the USSR but only recently has been investigated in the U.S.[2] As in Ref. 1, a properly formulated flame model for solid propellant combustion could have been used to calculate the heat feedback from the flame zone and surface reactions. However, a heat feedback function based on the Zeldovich approach was used here since it provides a more direct method of considering complex propellants in which the details of the flame zone and surface are not known quantitatively. (This is further explained in the following section.) This offers important advantages when considering propellants whose burning rate mechanisms are not understood (e.g., highly catalyzed propellants, propellants with complex r vs p relationships, and propellants with high fuel contents). However, we emphasize that before the method is applied, the validity of the assumptions should be evaluated at the particular conditions being studied.

A Matter of Correct Authorship

In Ref. 2 we describe a theoretical method for solving problems of nonsteady combustion and we named it the Zeldovich-Novozhilov method. After careful review of the literature, we believe we erred in giving it this dual name. Historically, the concept of treating a nonsteady solid propellant combustion wave as being made up of two parts, a quasi-steady reaction zone plus a completely nonsteady but nonreacting heat-up zone, joined together by a nonsteady heat feedback law, has so many independent authors that it would be improper to give the concept a particular name. However, the novel idea of by-passing the uncertainties of modeling a solid propellant flame and using instead the measurable steady-state burning characteristics $\bar{r}(p,T_0)$ to deduce the theoretically correct nonsteady feedback law $\phi(r,p)$ is due, as far as we can tell, to Zeldovich alone (1942).[3] In our opinion, this was a very clever idea, despite the limitations discussed in Ref. 2, and deserves recognition. The particular contribution of Novozhilov[4] to the work of Zeldovich consisted of finding the limiting conditions for steady-state burning (the intrinsic stability line) by employing this feedback law, but it is not used at all in the method we have described for analyzing sudden extinctions and overshoots. It is the Zeldovich method of deducing the feedback law (a sine qua non of all two-part quasi-steady theories) that is the heart of the present paper. Accordingly, we have decided to drop the earlier dual name and simply refer to the Zeldovich method for obtaining the feedback law $\phi(r,p)$. As a matter of further interest, when this function is inverted to form $r(p,\phi)$, it is called by Zeldovich and his colleagues the nonsteady burning rate law in contrast to $r(p,T_0)$, which is the conventional steady-state burning rate law.

Theory

Assumptions

The validity of the Zeldovich approach for calculating the heat feedback function in a particular combustion situation can be established only by analyses of the thermal reaction zones and

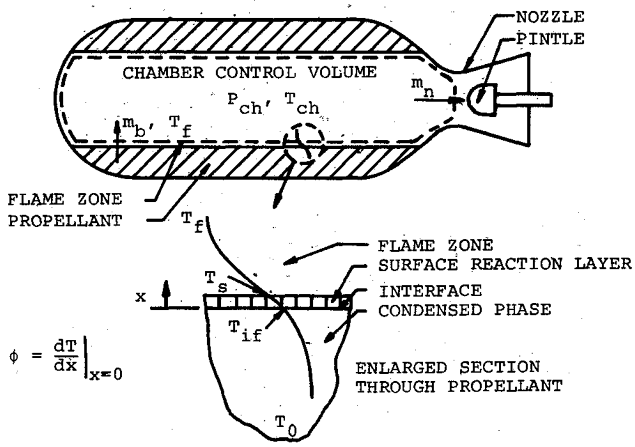

Fig. 1 Motor configuration and propellant zones considered in analysis.

the chamber flow conditions to determine the applicability of the assumptions. Figure 1 shows the motor and propellant control volumes considered in the analysis. The assumptions for the propellant are: 1) the rate processes in the gas phase and in the surface reaction zone can be considered quasi-steady in the sense that their characteristic times are short compared to the transition time of the pressure change (no such limitation is necessary for the thermal wave in the condensed phase); 2) no kinetic heat release occurs in the condensed phase below the surface reaction zone (although the surface reactions occur in a zone of finite thickness, the zone is sufficiently thin that it can be considered as quasi-steady); 3) the condensed phase of the propellant is accurately represented as being homogeneous and isotropic (e.g., for composite propellants the condensed phase thermal wave is larger than the mean diameter of the oxidizer particles); and 4) the propellant combustion zones are not influenced by axial position and external forces such as shear forces from the flowing chamber gases and acceleration.

The assumptions for the chamber are: 5) the axial variations of chamber temperature and pressure are negligible; 6) the chamber gases can be treated as perfect gases; and 7) heat losses to the inert parts are negligible.

The justifications for assumption 1 closely follow the arguments presented in Ref. 2. Thus, the characteristic times for the condensed phase, surface zone, and flame zone are

$$\tau_c = \alpha_c/r^2 \approx 0.01 \text{ sec for } r = 0.5 \text{ cm/sec} \qquad (1)$$
$$\approx 0.002 \text{ sec for } r = 3.0 \text{ cm/sec}$$

$$\tau_s \approx (RT_s/E_s)\tau_c \approx 0.1\tau_c < 0.001 \text{ sec} \qquad (2)$$

$$\tau_f = (\lambda_f c_c \rho_f / \lambda_c c_f \rho_c)\tau_c \approx 0.01\tau_c < 0.0001 \text{ sec} \qquad (3)$$

Since the characteristic times of the pressure excusion considered in the experiments herein are much longer than τ_f and comparable to τ_c, the conditions of assumption 1 are satisfied. The arguments for assumptions 2, 3, and 4 are the same as presented in Ref. 2.

In some situations in which metallized propellants are used, the quasi-steady assumption for the flame zone must be considered in terms of the reaction times of metals burning in the flame zone. For example, typical combustion times for metal agglomerates that form on the surface of some propellants[5] may be large compared to τ_f as calculated by Eq. (3). Also, the combustion of metal fuels such as aluminum may extend appreciably beyond the primary zone of gaseous reactions and the metal/metal oxide cloud may have appreciable inertia and thermal capacitance.[6] For conditions under which the propellant burning rate is influenced by the metal combustion rate, the validity of the Zeldovich heat feedback function requires that the times for the pressure excursions be long compared to the combustion times for the metal fuel.

The condition of negligible axial temperature variations of assumption 5 is more clearly justified for an internal-burning propellant grain with a low L^* than for an end-burning propellant grain with a large L^*. In the former case, the combustion gases leaving the burning propellant surface are distributed along the length of the rocket motor and have radial velocity components (normal to the axis) which promote rapid mixing. Also, in low L^* combustors the stay times of the combustion gases in the chamber are considerably less than the transition time for the pressure change, and as a result large axial thermal gradients cannot develop in the converging section of the nozzle. For example, the characteristic stay time of the datum case chamber (Table 1) is 0.0022 sec, while the monotonic portion of the pressure transition is 0.008 sec. The condition on axial pressure variations limits the Mach number along the propellant grain to values of less than approximately 0.3.

In applying assumption 5 to metallized propellants, it should be kept in mind that under some conditions metals burning in the chamber (i.e., away from the propellant surface) may influence the chamber gas to an extent comparable to the unsteady flow processes. Situations can be devised in which the reaction time of metals flowing along with the chamber gases is nearly equal to the stay time of the metal/metal oxide in the chamber. In such situations, the bulk mode analysis for the chamber, as presented in this paper, must be extended to account for dynamic heat release in the chamber.

Assumption 6 is widely employed in the combustion literature for the conditions considered in this paper. However, for completeness, the effect of the datum case pressure change (6.8–68.0 atm) was considered in terms of equilibrium thermochemical results.[7] Over the pressure range, γ_{ch} and M_w vary only 5% and 4%, respectively. The specific heat c_p variation between 0.46 and 0.925 affected the calculated p/p_{max} by less than 2%. Assumption 7 is consistent with the small amount of exposed motor hardware in properly designed high performance, internal burning systems.

Functional Relationships

The chamber pressure and mass discharge responses are calculated from a mathematical model whose primary components are coupled through numerical solutions to the energy equation for the propellant, the heat feedback function, and the energy and continuity equations for the chamber, which include the effects of throat area and chamber volume changes. The governing equations for the chamber are

initial equilibrium conditions at $t = t_1$

$$r = r_1;\ p_{ch} = p_{ch,1};\ T_f = T_f(r_1,p_1);\ T_{ch} = T_f;\ A_t = A_{t,1} \quad (4)$$

continuity equation

$$d(\rho_{ch}V_{ch})/dt + m_n = m_b \quad (5)$$

energy equation

$$d(\rho_{ch}V_{ch}T_{ch})/dt + m_nT_{ch} - m_bT_f - (1/c_p)d(p_{ch}V_{ch})/dt = 0 \quad (6)$$

Table 1 Datum case properties[a]

Propellant property	Datum value
Fuel rich (cool) propellant, 70% AP, 30% binder	...
Burning rate @ $p = 68$ atm, cm/sec	1.50
Exponent in burning rate law	0.50
Interface temperature @ $p = 68$ atm, °K	830.0
Flame temperature @ $p = 68$ atm, °K	1500.0
Propellant initial temperature, °K	298.0
Activation energy in pyrolysis law, cal/mole	20000.0
Burning rate temperature sensitivity, °K^{-1}	0.0020
Initial pressure, atm	6.80
Final pressure, atm	68.0
Duration of throat area change, sec	0.001
L^* for final throat, cm	117.0

[a] Activation energy is assumed independent of pressure.

chamber volume

$$dV_{ch}/dt = rA_b + \partial V/\partial t \quad (7)$$

where $\partial V/\partial t$ represents changes in free volume caused by actions such as insertion of the pintle.

flame temperature

$$T_f = (\lambda_c/\rho_c c_f)[\phi_{ref}/r_{ref} - \phi/r] - (T_{if,ref} - T_{if})c_c/c_f + T_{f,ref} \quad (8)$$

time-dependent conditions

$$A_t = f(t); \quad A_b = f\left(\int_0^t r dt\right) \quad (9)$$

The governing equations for the propellant are the heat flow equation for the nonreactive condensed phase

$$\partial T/\partial t + r(t)\,\partial T/\partial x = \alpha_c(\partial^2 T/\partial x^2) \quad (10)$$

with its boundary and initial conditions

$$T = T_0 \quad \text{as} \quad x \to -\infty; \quad \partial T/\partial x)_{if} = \phi(p,r) \quad (11)$$

$$T(x) = [T_{if}(r_1) - T_0]\exp(xr_1/\alpha_c) + T_0 \text{ at } t = t_1 \quad (12)$$

and the burning rate law in nonsteady form

$$r(t) = r[\phi(t),p(t)] \quad (13)$$

The factor ϕ is the desired heat feedback function obtained using the Zeldovich heat feedback function developed in Ref. 2 except that 1) it is based on T_{if} rather than a so-called burning-surface temperature (i.e., the temperature at the gas/burning surface interface) and 2) a general $r(p,T_0)$ relationship is used (i.e., σ_p does not have to be constant). This is a departure from the previous Zeldovich heat feedback formulation which required a constant σ_p. Once it is realized that the surface reaction zone is not a well-defined zone, it is more meaningful and less arbitrary to use T_{if} rather than a surface temperature. However, it is important that the region between x_{if} and the gas/burning surface interface be sufficiently thermally thin that assumption 1 applies. In this case of ammonium perchlorate (AP) composite propellants, the surface reaction zone is the region of AP intermediate decomposition products beyond the solid surface described in Ref. 8. On extinguishment, this layer has a frothly appearance. Analytical and experimental results (e.g., Ref. 9) have shown that in the range of pressures being considered, important burning-rate-affecting reactions below x_{if} are unlikely.

The nonsteady burning rate law can be deduced from measured steady-state burning rates as functions of pressure and initial temperature, as explained in Ref. 2. This means that the nonsteady law can be put implicitly in terms of exponent n and temperature sensitivity σ_p.

Equations (5–7 and 10) comprise a coupled set of three nonlinear first-order ordinary differential equations for changes in p_{ch}, T_{ch}, and V_{ch} and one second-order partial differential equation for T_c. Equation (10) with its initial and boundary conditions was solved using the methods of explicit finite differences. Equations for p_{ch}, T_{ch}, and V_{ch} were integrated by a fourth order Runge Kutta method using the time step that satisfied the stability criterion for the condensed phase energy equation. For special limiting cases, the numerical results agreed with closed form solutions for chamber venting, adiabatic compression, steady-state burning, etc. A similar set of equations were formulated in the USSR; however, a closed formed solution is still being sought.[10]

Experimental Method

Transient pressure data (from which the dynamic burning rates can be determined) were obtained by suddenly reducing the nozzle throat area of a motor with a rod-and-tube propellant charge (Fig. 2) designed to have a very small free

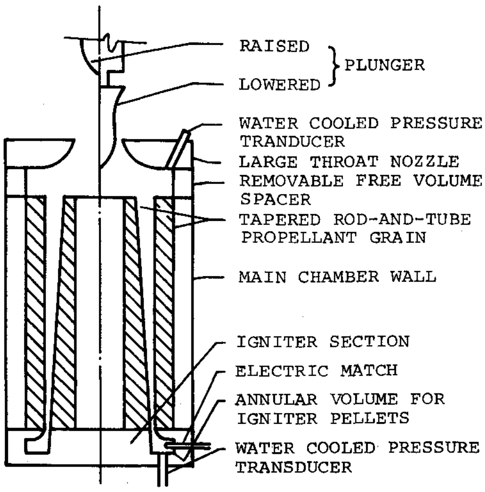

Fig. 2 Low L^* combustor used to produce rapid pressure and burning rate transients.

volume in order to have a rapid pressure rise. (This is referred to as the low L^* combustor.) After ignition, the motor is allowed to reach equilibrium at low pressure, and then the throat area is rapidly reduced (as fast as 0.001 sec) by insertion of a pintle driven by a pneumatic cylinder. The chamber pressure rises sharply to a new equilibrium level. This method of obtaining rapid pressure changes and dynamic burning rate data has the potential of producing a more easily interpreted result than previously attempted methods. For example, secondary powder charges have been used to pulse combustors (e.g., Ref. 11). Establishing the dynamic burning rate from such experiments is complicated by the impulsive changes brought on by the secondary charge. Similarly, obtaining dynamic burning rate information from T-burner experiments is complicated by approximations of the losses in the T-burner.[12] Also, it is not feasible to deduce dynamic burning rate information from the rapid pressure rise following ignition since the complexities of ignition processes[13] preclude isolating the desired dynamic burning effects.

Results

Figure 3 shows measured rapid pressurization and overshoots (in the case of tests 2 & 3) resulting from the dynamic super-rate effects for a sudden throat area decrease (in 0.001 sec). Although analysis of the data reveals that test 1 also exhibits a strong super-rate effect, the overshoot of pressure effect is muffled by the somewhat larger L^* associated with the smaller final A_t. The propellant was 70% AP (30% 15μ and 70% 45μ), 29% PBAA/EPON and 1% CuO/Cr_2O_3. Other motors have been tested over a range of chamber conditions. The instantaneous burning rates and chamber temperatures were deduced from the measured p vs t by finding T_{ch} vs t and r vs t which satisfy the energy and continuity equations for the chamber. Deducing T_{ch} and r from measured p vs t is for a much simpler situation than the one described in the section entitled Theory since in deducing T_{ch} and r from measured p vs t, flame temperature (not the chamber temperature) is assumed to be constant and the partial differential equation governing heat flow to the condensed phase is not solved. Figure 4 shows the deduced values of dynamic burning rate, r/r_{eq}, deduced from the measured p vs t of tests 1 and 2. In Fig. 5, r/r_{eq} is displayed in terms of a commonly used dimensionless $\dot{p}$ parameter.[14] The dynamic burning rates of Fig. 5 demonstrate a hysteresis-like character rather than a simple straight-line relationship between the dimensionless $\dot{p}$ and dynamic burning rate.

The solution described in the section entitled Theory was used to predict the p vs t in the test firings (Fig. 6) and to investigate the effects of motor parameters on variations of p_{ch}, r, T_{ch}, and T_f. For the particular configuration used in our tests, the small reduction in V_{ch} resulting from the pintle insertion did not affect appreciably p_{ch} vs t. The function $\phi(p,r)$ is calculated using the approach described in Ref. 2 and the burning rate data of Table 1. For the propellant and the range of pressures being considered, the measured parameters n and σ_p are constant. Figure 7 shows calculated results for the transition from steady state at pressure p_1 to another pressure p_2 produced by a ramp change in throat area from $A_{t,1}$ to $A_{t,2}$ in 0.001 sec. Figures 8 and 9 show the effect of burning rate exponent and temperature

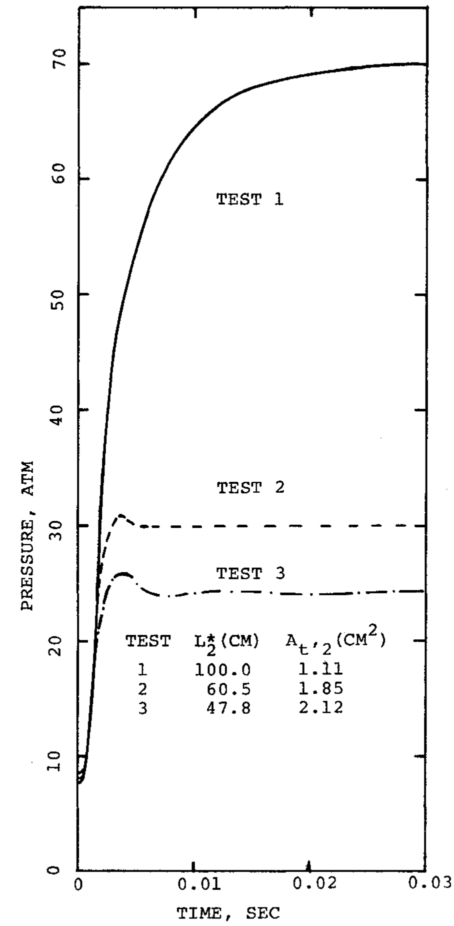

Fig. 3 Measured rapid pressurizations and pressure overshoots resulting from rapid decrease in throat area.

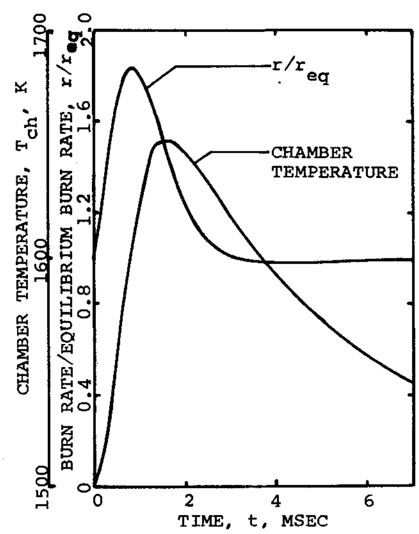

Fig. 4 Dynamic burning rate and chamber temperature overshoot deduced from test 1.

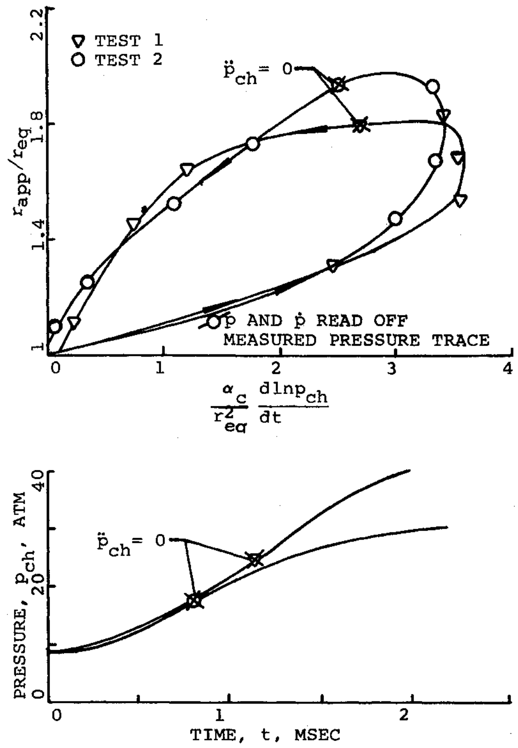

Fig. 5 Experimentally determined burning rate correlated with a dimensionless *p*.

sensitivity of burning rate on the relative magnitude of the dynamic burning super-rate and dynamic pressure overshoot during the pressurization. These figures illustrate one of the advantages of our approach in that the results can be presented in terms of readily measured propellant parameters.

Conclusions

The analytical and experimental results presented here have demonstrated the influence of dynamic burning and dynamic chamber effects on the pressure response of rocket motors experiencing rapid reduction in throat area. The analytical model was shown to be successful in predicting the general pressure response of an experimental motor undergoing such area changes and in determining the influence of motors experiencing similar dynamic changes. The analysis has shown that high pressure exponent, high temperature sensitivity, high dA_t/dt, low burning rate and low L^* all tend to increase the dynamic effects. However, the degree of success of the Zeldovich heat feedback function in predicting the extent of dynamic burning rate effects depends heavily on the accurate knowledge of propellant properties such as $\sigma_p(p,T_0)$ and $T_{if}(r,T_0)$ and on the characteristic times of the several flow and combustion processes.

The variable throat area, low L^* combustor has demonstrated its usefulness in obtaining pressure time data from which dynamic burning rate and chamber temperatures can be determined. Studies are presently being conducted to determine whether the combustor can be used to provide a more direct

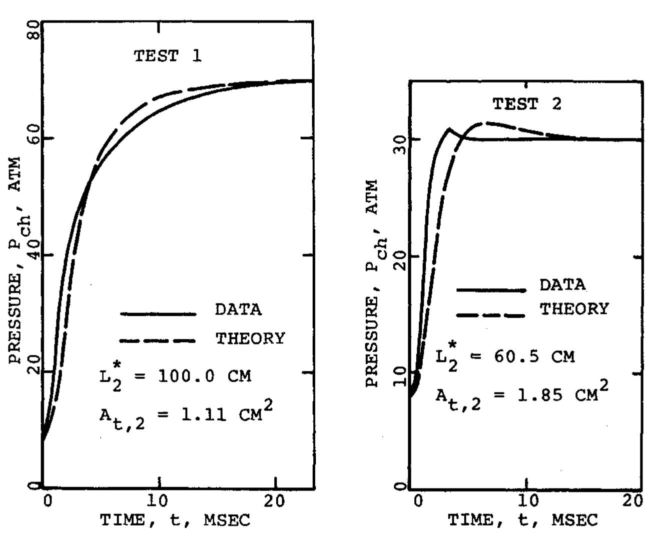

Fig. 6 Calculated and experimental pressure-time traces.

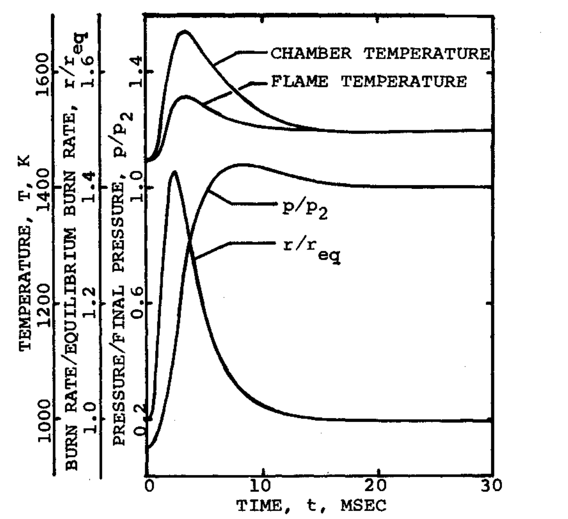

Fig. 7 Calculated pressure, chamber temperature, flame temperature and burning rate for datum case.

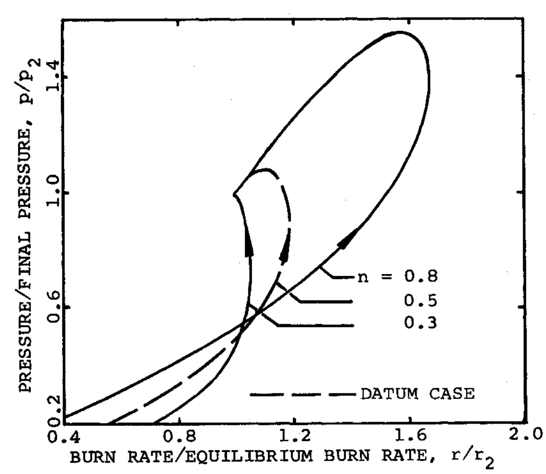

Fig. 8 Influence of burning rate exponent.

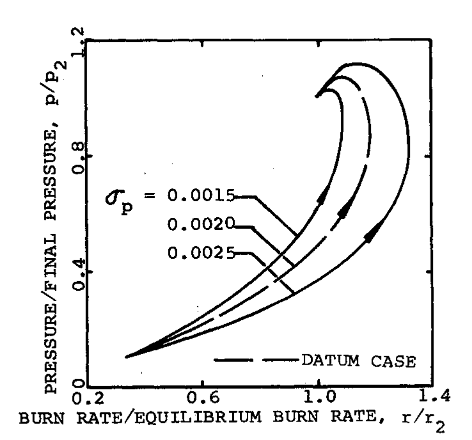

Fig. 9 Influence of temperature sensitivity.

measurement of admittance and stability limits than can be obtained using T-burner results.

The results of this study, therefore, have implications that go far beyond the measurement and prediction of dynamic burning rate and pressure effects in a variable area, low L^* motor. The principles developed in the study can be applied to a broad range of transient combustion situations, such as L^* combustion instability, throttleable rocket motors and closed chamber experiments.

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